held to be more than rules for valid inference. As Spinoza put it, "The order and connection of ideas is the order and connection of things." The laws were thus deemed to be laws of being as well as laws of thought, to be constitutive principles of metaphysics as well as of logic. The proposition connects a subject with a predicate because the world consists of substances that have attributes. This was a stunning insight; we will shortly see whether it can be maintained.

Criticism of Traditional Logic

Kant wrote in 1787 that logic, since Aristotle, "has not been able to advance a single step, and is to all appearances a closed and completed body of doctrine." But soon thereafter, in a remarkable intellectual explosion, the two-thousand-year-old structure of logic was torn apart by such men as Boole, DeMorgan, Frege, Peano, Peirce, Schiller, Russell, and Whitehead. There were three major lines of critical analysis.

I. Essentially, Aristotle's view of propositional form was found to be insufficiently precise. He treated alike all propositions of the form "all x is y," although this form may actually cloak some sharply different kinds of assertion. Some examples are:

1. "All bachelors are unmarried men." Definition.
2. "All whales are mammals." Analytic inclusion of a subclass within a class.
3. "All cabinet members are Republicans." Inductive inclusion of a subclass within a class (based on the enumeration of a finite class—in this case the class of cabinet members).
4. "All politicians are liars." Synthetic inductive generalization based on a sample, believed probably correct.
5. "All full professors are Ph.D.'s." Statement of a requirement, or resolution: in order to be a full professor, one must possess a Ph.D.
6. "All citizens of authoritarian dictatorships who openly oppose the regime are brave." Hypothetical statement that if indeed there are any such citizens, they would be brave; but there may be none.

Note also the variety of meanings that may be contained in the copula *is*:

1. "St. Anne is the mother of Mary." Identification.
2. "The raven is black." Ascription of an attribute to a subject.
3. "Ted Kennedy is a senator." Assertion of membership in a class (in this case, the class of senators).
4. "To strive is to succeed." Assertion of entailment: if you strive, then you will succeed.
5. "God is." Assertion of existence.
6. "This spoon is silver." Description of composition: the spoon is *made up of* silver.

(Russell called the ambiguity of *is* "a disgrace to the human race." That ambiguity is the reason why metaphysical references to "Being" are often profoundly confused.)

Aristotle's propositional forms were thus found to be less satisfactory for the organization of knowledge than had been supposed (although the validity of his logic is not thereby denied).

II. A second deficiency of traditional logic is its failure to elucidate some important aspects of inference, especially the logic of relations and the sentence connectives. Thus:

1. Such propositions as "Socrates is the teacher of Plato" do not lend themselves to the connection of a subject with a predicate by the copula *is*; rather, the terms "Socrates" and "Plato" are connected by the relation "is the teacher of." The proposition "New York is between Boston and Washington" connects three terms by the relation of "betweenness."
2. *Sentence connectives* are such terms as *and, or, not,* and *if . . . then . . .* Consider the ambiguity of *and* in "I love apples *and* oranges" and "I love peaches *and* cream." Or consider the implicit temporality in "They got married *and* had children" and "They had children *and* got married." The word *and* operates differently in these examples. Consider the ambiguity of *or* in "Take a raincoat *or* an umbrella" and "This

medicine will cure you *or* it will kill you." The former (inclusive) *or* permits both alternatives; the latter (exclusive) *or* does not. (When G. E. Moore's wife gave birth, a student asked, "Professor Moore, is it a boy or a girl?" Moore chortled, "Yes!") Relations and sentence connectives are clarified in modern logic.

III. A third line of criticism centered upon the three "laws of thought." What kind of laws are they, exactly? And what is it that gives them their special character, their aura of absolute undeniability? Are they based on how people actually think? But many prominent citizens seem to have no difficulty in thinking self-contradictory thoughts. And, in any event, to say that the laws of thought reflect thinking processes would be to confuse empirical descriptive psychology with logic, which is normative, and which considers not the process of thinking but the propositions that result from this process. (Schiller made this mistake.) If the laws of thought, then, are rules that prescribe how people *ought* to reason, why ought they do so? To say that someone ought to do something implies that he might not do so; for instance, if you tell me that I ought to tell the truth, it is because I might lie; but how could I possibly violate the law of identity?

Are the three laws perhaps based on the requirements of language, as some philosophers have asserted, so that all that logic reveals is grammar or syntax? But language and grammar have been found to be mutable and diverse in ways that distinguish them clearly from logic. Are the three laws learned from experience, just as we learn the most general laws of science? (Mill argued for this empiricist interpretation.) But there is no evidence at all for such learning, and, unlike the laws of science, there is no sense in which the laws of thought can be found false.

Are the three laws all that are needed for valid inference, which might account for their special status? No, the more thoroughgoing analysis of modern logic indicates that there are other rules that require explicit statement; for example, *permutation* (if p and q are propositions, then "p or q" is equivalent to "q or p"), and *modus ponens,* or detachment (if p is true, and p implies q, then q is true).

Most perplexing, perhaps, is the question of whether and how

the three laws can be laws of being as well as laws of logic. The term "being" is, at best, misleading and bewildering; at worst, almost meaningless. The pervasive ambiguity of the verb "to be" and the confusions involved in what has been called "the ontological commitment of the copula" (even though many languages—Hebrew, for example—have no word for *is*) have yielded some staggering metaphysics and theology. In addition, what could it mean to say that the laws of identity, excluded middle, and contradiction apply to the world: is the world in fact the kind of place in which things always are unchanging and retain their identity? or in which everything is unambiguously either wet or not wet? and never both wet and not wet?

The New Logic

The new logic is motivated by the desire for greater clarity, precision, completeness, generality, and utility. It is expressed in symbols, and embodied in systems which state explicitly what is assumed, and what is inferred, and how. The triumphant climax of the revolution in logic was the *Principia Mathematica* of Whitehead and Russell, the first volume of which appeared in 1910. They employed two "primitive" connectives, the curl $\sim$ (for *not*) and the wedge $\vee$ (for *or;* it permits the truth of either or both alternatives). By means of these two primitives, Whitehead and Russell defined the three symbols for *if . . . then . . .* (the horseshoe $\supset$), *and* (the dot $\cdot$), and equivalence (three lines $\equiv$). They specified two rules of inference or derivation (substitution, and *modus ponens*) and five primitive (i.e., unproved) propositions: permutation, tautology, addition, association, and summation. (The symbolic apparatus was later simplified: all the connectives, for example, were found to be reducible to a single primitive—Scheffer's stroke function for alternative denial; and all the five primitive propositions to one; but these are technicalities.) Whitehead and Russell's system is so explicit and exhaustive that the proposition "$1 + 1 = 2$" is not proved until well into the second volume.

One consequence of this new insight is the divorce of logic from metaphysics. Logic is now seen as a set of rules, or calculus,

by which propositions are transformed into, or inferred from, other propositions. Nothing, no new item of knowledge, is added thereby. Logic operates like a meat grinder or juice extractor: the output differs from the input, but contains nothing new. In this extensional, or *truth-functional,* view of logic, the truth of any complex or inferred proposition depends only on the truth of its ingredients, or premises. Logic is not concerned with meanings, or with facts, or with Being, but only with validity of inference. Logic uses "meaningless" symbols in order to concentrate on form alone: it sets norms only because it exposes forms; that is, it tells you what the consequences are of using symbols in certain ways. Logic is autonomous. It elucidates what a proof is. The virtue of the symbolism is its perspicuity; that is, that you can see through it clearly. There is no "objective correlate" or "residual essence" to the symbols used for the transformation of propositions; nothing in the world is controlled by the rules of *modus ponens* or equivalence or negation. Gilbert Ryle makes this point forcefully:

> The world neither observes nor flouts the rules of inference any more than it flouts the rules of bridge, prosody, or viticulture. The stars in their courses do not commit or avoid fallacies any more than they revoke or follow suit.

Logic is "true in all possible worlds" because it shows what is meant by the term "possible": a world in which "$2 + 2 = 5$" is not a possible world.[1]

It is important to realize that the horseshoe symbolizes *material implication,* not necessity. The proposition "if p, then q" or "$p \supset q$" is equivalent to the proposition "either p is false or q is true" and to the proposition "it is not the case that p is true and q is false." If the proposition "$p \supset q$" is true, it is because of the truth values of p and q separately; that is, the truth of the complex proposition is a function of the truth of the simple propositions which it combines. As we noted in our discussion of causality in Chapter 1, philosophers since Hume have doubted whether there is any necessary connection between two events which are described as cause and effect. There is no logical necessity that

[1] An old riddle asks, how far can a dog run into a forest? The answer is, halfway, because after that he's running out of the forest. This is not a truth about the world, but about the terms *into, out of,* and *halfway,* which we have devised.

the sun rise tomorrow, nor that stones fall. The new truth-functional, or extensional, logic thus dissolves necessary implication into material implication.

Inference and Implication

If logic is a meat grinder, and produces nothing that was not in the premises, do we then never acquire new knowledge from it? Learning is indeed temporal, although logic is not. We acquire knowledge by making inferences; this is psychological, and takes time. But if the inference is valid, it is because it accords with logical implications, and these are not new but timeless. Nevertheless, in pursuing logical analysis our discoveries may in fact be rather startling. John Aubrey describes how Thomas Hobbes first came by chance upon a copy of Euclid, lying open at the page containing Pythagoras' theorem.

> He read the proposition. "By God," sayd he, "this is impossible." So he reads the demonstration of it, which referred him back to such a proposition; which proposition he read. That referred him back to another, which he also read . . . at last he was convinced. . . . This made him in love with geometry.

In another elegant proof, Euclid showed that there is no greatest prime number.[2] This may be new knowledge to you, even though it is timelessly true.

In a recent Broadway play, a guest at a party asks a Catholic priest, "Don't you sometimes hear embarrassing things in confession?" "Oh, yes," he replies, "in fact when I was just starting out as a young priest, the first man who came to me for con-

[2] A prime number is not divisible by any number except by itself and 1. If you were to try to list all the prime numbers, you would find that the higher you go, the farther apart they are, since the more numbers you pass, the more frequent become their multiples. It seems plausible, then, that eventually there might be a last or greatest. But Euclid proved that the notion of a greatest prime number is self-contradictory: whatever prime number n you might assert to be the greatest, if you were to multiply it by every number smaller than itself, and then add 1, you would then get a number $(n!+1)$, which would of course be greater than n. If this new number is prime, then n cannot be the greatest prime; and if this new number is not prime, then its prime factors (if it has any) must be greater than n, since it is not divisible by n or by any number smaller than n, because you added 1 to your product.

fession told me he had committed a murder." Later on in the play, a newcomer joins the party, and, on being introduced to the priest, says, "I met you long ago, Father; in fact I was the first man to come to you for confession." The audience gasps, suddenly realizing that the man is a murderer. But that inference to new knowledge is timelessly implied by the logic of the premises.

An ancient rationalist tradition in philosophy seeks to discover the "first principles" of nature and then to arrive by reason alone —as Euclid's geometry did—at precise, intersubjective, eternally true propositions about the world. Pythagoras sought these first principles in numbers as the substructure of reality; Plato and Aristotle relied on intellectual intuition to arrive at them; Plotinus on mystical ecstasy; Descartes on clear and distinct ideas. Indeed, many impressive advances in knowledge have come about by attempting to "reason out" the structure of things. Thus, Mendeleev in 1869 sorted out the sixty-three then known chemical elements into a "Periodic Table" according to their atomic weights and chemical properties. In order to make the pattern of his Table "come out right" Mendeleev had to adjust some of the weights and leave some gaps, yet he predicted that the gaps would be filled by elements whose existence was then unsuspected; and indeed those elements have since been found. Bode's Law (about 1770) predicted correctly the discovery of unknown planets in our solar system, based on the mathematical relations of their orbits.

But alas for the view that reason alone can provide knowledge of the world! Let me cite merely two of many rationalist proofs that misfire. One Francesco Sizzi, arguing against Galileo's discovery of the satellites of Jupiter, pointed out that

> There are seven windows in the head, two nostrils, two eyes, two ears, and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena in Nature, such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven.

And the great Kepler in 1596 insisted that

> God in creating the universe and laying out the heavens had in view the five regular solids of geometry, celebrated since the time of Pythagoras and Plato, and that it was in accordance with their properties that he fixed the number of the heavens, their proportions, and the ratio of their movements.

It now seems quite clear that logic and mathematics deal, as Peirce says, with "hypothetical states of things" and "assert no matter of fact whatever." Or Einstein: "in so far as mathematics is about reality, it is not certain, and in so far as it is certain, it is not about reality." Or Russell: "Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true."

Logic as a Regulative Calculus

There were three baseball umpires, calling balls and strikes. The first one said, I call them as I see them. The second one said, I call them as they are. But the third one declared, until I call them, they ain't nothing. The first umpire is a sense datum subjectivist: nothing will persuade him that he may have made a mistake. The second is a rationalist: he declares that he is describing the objective structure of the world. Only the third realizes that sorting out pitched baseballs is a man-made ordering; nothing in the world is objectively a "ball" or a "strike" without an umpire's decision.

But even though the rules of logic do not describe the world, this does not mean that they are conventional, like the rules of baseball or bridge. Conventions (such as driving on the right side of the road) may always be altered; but there is no alternative to logic. If we changed the postulates of *Principia Mathematica,* we would produce not a different logic, but either another version of the same logic, or no logic at all. Postulates and theorems may in fact often be interchanged; but "the deviant logician's predicament," says Quine, is that "when he tries to deny the doctrine, he only changes the subject." There are indeed different kinds of logic—multi-valued logics and modal logics (which deal with possibility and necessity). But these are not alternatives: they merely add to logic certain undefined terms. There is no alternative to logic, as there is to baseball or to bridge, because logic uniquely serves the human need to under-